Data C100, Midterm Exam

Summer 2024

Name: _____

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Student ID: _____

Name of the student to your left: _____

Name of the student to your right: _____

Instructions:

Do not open the exam until instructed to do so.

This exam consists of **72 points** spread out over **6 questions** on **21 pages** and must be completed in the **110 minute** time period on July 19, 2024, from 9:10 AM to 11:00 AM unless you have pre-approved accommodations otherwise.

Note that some questions have circular bubbles to select a choice. This means that you should only **select one choice**. Other questions have boxes. This means you should **select all that apply**. Please shade in the box/circle to mark your answer.

There is space to write your student ID number (SID) in the upper right-hand corner of each page of the exam. **Make sure to write your SID on each page** to ensure that your exam is graded.

Honor Code [0pt or $-\infty$]:

As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. I am the person whose name is on the exam, and I completed this exam in accordance with the Honor Code.

Signature: _____

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1 Ain't No Free Chocolate Milk [13 Points]

Aditi and Zekai are interested in conducting market research on the chocolate milk drinking habits of UC Berkeley students. More specifically, they want to find out what the most popular chocolate milk is among above-average spenders. With the limited funds at hand, they decide that they can only survey a small sample of students. They devise several different sampling schemes together.

(a) [2 Pts] What is the *population of interest* in this study? *Use no more than 15 words.*

(b) [4 Pts] Aditi suggests that all TAs in Data 100 select every fifth student in the order that they show up to discussion sections this week and ask them for their favorite chocolate milk brand.

(i) What is the *sampling frame* of this method? *Use no more than 15 words.*

(ii) Which of the following statements are true about Aditi's sampling scheme? **Select all that apply**.

- ☐ This sampling scheme is a probability sample whereby everyone in the sampling frame has the same probability of ending up in the sample
- ☐ This sampling scheme is not a simple random sample, as not every pair of students in our sampling frame has the same probability of ending up in our sample.
- ☐ It is possible that a sampled student is in the sampling frame and is in the population of interest.
- ☐ It is possible that a sampled student is not in the population of interest and is not in the sampling frame.
- ☐ None of the above

(iii) Which types of biases could occur with this sampling scheme? **Select all that apply**.

- ☐ Selection Bias
- ☐ Non-response Bias
- ☐ Response Bias
- ☐ None of the above

(c) [3 Pts] Zekai proposes a different sampling scheme. Instead of selecting every fifth student in order of arrival, he proposes that TAs randomly pick one student out of the first seven students that arrive to their section. The TA then repeats this process for the next seven students.

For instance, for the first seven students that arrive, a TA randomly selects one student. Among the next seven students that arrive, they randomly select one student, and so on.

(i) What is the probability that a student who attends discussion this week ends up in our sample? Assume that 35 students show up.

(ii) What kind of sample is this? **Select all that apply**.

- ☐ Probability Sample
- ☐ Simple Random Sample
- ☐ Deterministic Sample
- ☐ None of the above

Aditi and Zekai happen to have a friend who works at a company that makes software that tracks customer purchases for chocolate milk manufacturers. This friend gives them a dataframe `choco_sum` of which the first rows are displayed below. The dataframe contains a row for each above-average spending Berkeley student (tracked through their berkeley.edu email), along with the total amount spent on chocolate milk during 2023. The `email` column contains `str` values. `amount` consists of `float` values.

	email	amount
0	aditi@berkeley.edu	105.0
1	charlie@berkeley.edu	25.0
2	james@berkeley.edu	35.0
3	student1@berkeley.edu	500.0

We have another dataframe `survey` that contains the results from our survey. The table has one row for each student that completed the survey according to the sampling scheme in (c). Here are the first couple of rows:

	email	favorite brand
0	student1@berkeley.edu	Fairlife
1	student2@berkeley.edu	Nestlé
2	student3@berkeley.edu	Horizon
3	student4@berkeley.edu	Hershey's

1. `email` (str): The student's Berkeley email
2. `favorite brand` (str): The student's favorite chocolate milk brand

(d) [2 Pts] Finish the blanks below such that the dataframe, `choco_survey`, contains one row for each above-average spending student who filled out the survey. The dataframe should contain three columns: `email`, `amount`, and `favorite brand`.

```
choco_survey = _____(A)_____
```

Fill in the blank `(A)`

(e) [2 Pts] Aditi and Zekai decide to resample from their original sample *with replacement* to get an understanding of the uncertainty. What is the probability that a resample of size 20 contains exactly 1 student whose favorite brand is Fairlife **and** exactly 1 student whose favorite brand is Horizon? The proportions of favorite brands in our original sample is shown below:

	Favorite Brand	Proportion
0	Hershey's	0.30
1	Nestlé	0.50
2	Fairlife	0.15
3	Horizon's	0.05

Leave your answer as a mathematical expression.

2 Godly Skincare [12 Points]

Dan is scrolling through BikBok one day and discovers the importance of skincare. He starts using products he found online and keeps track of these products in a `DataFrame` called `dan_products`. The columns are as follows:

- `Product`: The name of the product (type = `str`).
- `Brand`: The brand the product is from (type = `str`).
- `Aura`: The effectiveness of a product according to Dan (type = `numpy.int64`).
- `Price`: The price of a product in USD on Berkazon (type = `numpy.float64`).
- `Stars`: The online rating of the product on Berkazon (type = `numpy.float64`).
- `Repurchased`: Whether or not Dan has repurchased the product (type = `numpy.bool_`).

(a) [1 Pt] Dan can't remember what `Aura` he gave the "Lip Glowy Balm" from the brand "Laneige". Fill in the blank below so that `glowy_aura` is the `Aura` integer value for this product.

`glowy_aura = _____(A)_____`

Fill in the blank `(A)`

(b) [1 Pt] Dan tells you that 1) he has sorted `dan_products` so that `Aura` is in descending order and 2) that there are 719 rows in the `DataFrame`. Fill in the blank below so that `med_stars` is the median `Stars` value. You are **not** allowed to use `np.median`.

`med_stars = _____(A)_____`

Fill in the blank `(A)`

(c) [2 Pts] Dan wants to know the average `Aura` AND maximum `Stars` for each brand. Fill in the blank below so that `brand_stats` is assigned to a `DataFrame` that can represent this.

`brand_stats = _____(A)_____`

Fill in the blank `(A)`

(d) [3 Pts] Dan wants to give Xiaorui a great product so that he can begin his K-Drama arc, but he doesn't want to give Xiaorui products from brands he doesn't like. Select the following option(s) (**select all that apply**) that can return the name of the product with the highest `Stars` that comes from a brand with an average `Aura` greater than or equal to 3.5. Assume that there are multiple brands in the `DataFrame` that have an average `Aura` greater than or equal to 3.5.

☐
```
dan_products.groupby("Brand").filter(lambda x: x["Aura"]\
.mean() >= 3.5).sort_values("Stars", ascending = False)\
["Product"][0]
```

☐
```
dan_products.groupby("Brand").filter(lambda x: x["Aura"]\
.mean() >= 3.5).sort_values("Stars", ascending = False)\
["Product"].iloc[0]
```

☐
```
dan_products.filter(lambda x: np.mean(x["Aura"]) >= 3.5)[0]
```

☐
```
dan_products.groupby("Brand").filter(lambda x:\
np.mean(x["Aura"]) >= 3.5).sort_values("Stars",\
ascending = False).iloc[0]["Product"]
```

☐ None of the above.

(e) [2 Pts] Dan wants to find out if a product being repurchased has a greater amount of `Aura`. Fill in the blank below to create a `DataFrame` called `repurch_aura` that finds the average `Aura` for each brand, separating it into two columns on whether or not a group of products has been repurchased. If there's a category with no data, keep the value `NaN`.

`repurch_aura = _____(A)_____`

Fill in the blank `(A)`

(f) [3 Pts] Dan is browsing Berkazon and comes across a review from a skincare influencer that he really likes.

```
review = "This was the WORST (!!!)  thing EVER!!  i only liked the
CUT3 pets on the packaging.ZZZ..."

mysterypattern = r"([A-Z!]+)[^\.]"

re.findall(mysterypattern , review)
```

Select all the strings that are returned in the call to `re.findall` above (**select all that apply**). Note that spaces are represented as an underscore.

☐ `"CUT3"` ☐ `"ZZZ"`

☐ `"EVER!!"` ☐ `"T"` ☐ None of these strings.

☐ `"WORST_"` ☐ `"!!!"`

3 Spill the Bubble Tea [8 Points]

Angela loves TPTea and has taken to exploring the different drink options available. She compiled this data in a DataFrame called `boba`. Below you can find the descriptions of its columns and the rows. We have also provided the first couple of rows.

- `drink`: name of the boba drink (type=`str`)
- `price`: price of the boba drink (type=`numpy.float64`)
- `bestseller`: whether the drink is a bestseller drink or not (type=`numpy.bool_`)
- `num_times`: the number of times Angela has gotten the drink (type=`numpy.int64`)
- `rating`: Angela's rating for the drink between [1, 10] (type=`numpy.int64`)
  - `1`: "I would never get this drink again"
  - `10`: "This is my go-to order"

	drink	price	bestseller	num_times	rating
0	Tie Guan Yin Milk Tea	5.95	False	10	10
1	Boba Milk Tea	5.95	True	4	8
2	Strawberry Milk Tea	6.75	False	6	9
3	Passion Fruit Green Tea	6.25	False	1	4
4	Peach Fantasy Ice Tea	6.50	False	0	5
5	Mango Cheese Cream Foam	6.95	True	5	10
6	Thai Milk Tea	5.75	False	1	2
7	Winter Melon Lemon	5.99	False	0	5

(a) [2 Pts] Angela wants to perform exploratory data analysis on the dataset. To familiarize herself with the data, she first identifies the variable types of the columns in `boba`. Determine the variable type that best describes each of the following columns in `boba`.

(i) `drink`

- ◯ Qualitative Nominal
- ◯ Qualitative Ordinal
- ◯ Quantitative Continuous
- ◯ Quantitative Discrete

(ii) `price`

- ◯ Qualitative Nominal
- ◯ Qualitative Ordinal
- ◯ Quantitative Continuous
- ◯ Quantitative Discrete

(iii) `bestseller`

- ◯ Qualitative Nominal
- ◯ Qualitative Ordinal
- ◯ Quantitative Continuous
- ◯ Quantitative Discrete

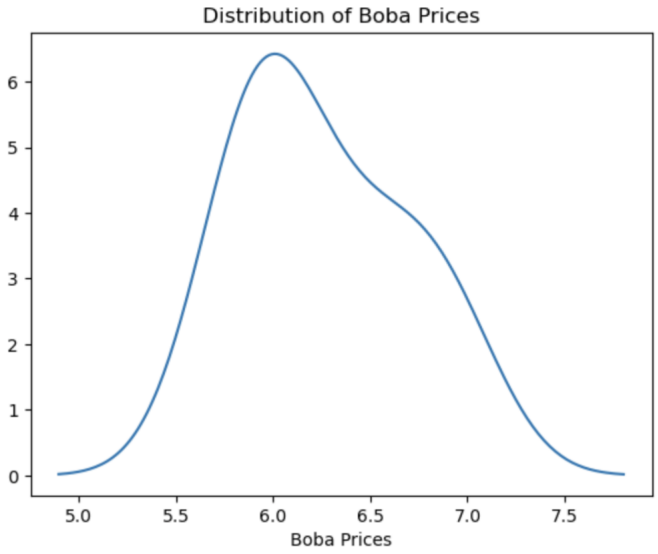
(iv) `num_times`

- ◯ Qualitative Nominal
- ◯ Qualitative Ordinal
- ◯ Quantitative Continuous
- ◯ Quantitative Discrete

(b) [2 Pts] Which of the following are possible visualizations for `price` only? **Select all that apply.**

- ☐ Bar chart
- ☐ Histogram
- ☐ Box plot
- ☐ Line Plot
- ☐ Contour Plot
- ☐ None of the Above

(c) [2 Pts] Angela wants to create a KDE curve of the distribution of boba prices in the dataset `boba`. She generated the following KDE curve.

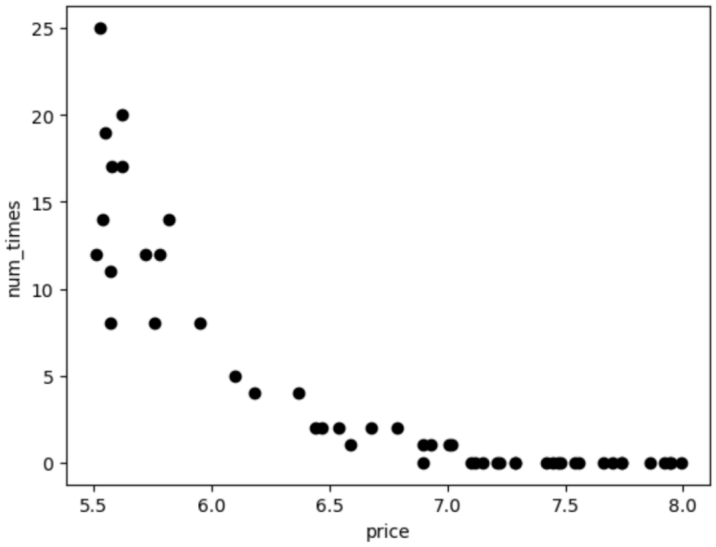


Given the plot above, is this curve a valid KDE curve? **Select one.**

- ◯ Yes
- ◯ No
- ◯ Not enough information

Explain your answer to the previous part. No credit will be awarded to the previous part if no explanation is provided. *Do not use more than 15 words.*

(d) [2 Pts] Angela collects more data on the other drinks offered at TPTea from the students in her section. She decides to plot the relationship between `price` and `num_times` and gets the following plot:



Which transformations should Angela apply to make it so that the relationship between `price` and `num_times` is linear? **Select all that apply.**

- ☐ $\sqrt[3]{x}$
- ☐ x^3
- ☐ $\sqrt{y}$
- ☐ $\log(y)$
- ☐ None of the above

4 Lotsa Loss [13 Points]

Suppose we have a dataset with n datapoints and one input, $\{(x_i, y_i)\}_{i=1}^n$, where $x_i \in \mathbb{R}$ is the input and $y_i \in \mathbb{R}$ is the target output. We want to use regression on the inputs x_i to predict y_i. Boyu suggests we use the constant model:

$$\hat{y}_i = \theta \text{ where } \theta \in \mathbb{R}.$$

Additionally, he has a new loss function for us, Weighted Mean Squared Error (WMSE). This loss function allows us to up- and down-weight datapoints. For example, perhaps we want to assign higher weights to more recent datapoints. The formula for WMSE is as follows:

$$\text{WMSE} = \frac{1}{n}\sum_{i=1}^{n} w_i(y_i - \hat{y}_i)^2$$

(a) [2 Pts] Say we have the following table which includes our datapoints and the weights we have assigned them:

i	x_i	y_i	w_i
1	100	10	0.5
2	50	20	1.0
3	30	30	1.5
4	20	10	1.0

Calculate the weighted WMSE for $\theta = 20$. **Draw a box around your final answer.**

(b) [3 Pts] Take the derivative with respect to θ of the WMSE of the model. **Draw a box around your final answer and be sure to simplify it in terms of** $x_i, y_i, \theta, w_i,$ **and/or** n.

(c) [2 Pts] Now, calculate the value of $\hat{\theta}$, which minimizes the Weighted Mean Squared Error (WMSE) of our model. You may assume, without proof, that the critical point of this loss function is guaranteed to be a minimum. **Draw a box around your final answer and be sure to simplify it in terms of** $x_i, y_i, \theta, w_i,$ **and/or** n.

(d) [2 Pts] Which of the following statements are true about the model and the loss function defined above? **Select all that apply.**

- ☐ The model assumes that all observations have the same variance.
- ☐ The WMSE gives more importance to observations with higher weights.
- ☐ The sum of the residuals is always zero.
- ☐ The weighted sum of the residuals is always zero.
- ☐ For a fixed set of weights, there is always a single unique optimal value for $\hat{\theta}$ for a given dataset.
- ☐ None of the above.

(e) [2 Pts] Let's now think about Mean Squared Error (MSE). Recall that MSE is defined as follows:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

Which of the following functions are appropriate loss functions that penalize outliers more harshly than MSE? **Select all that apply.**

- ☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y})$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}[|y_i - \hat{y}_i|^3 + (y_i - \hat{y}_i)^2]$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^4$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^3$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^5$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|^3$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}|(y_i - \hat{y}_i)^5|$
- ☐ None of the above.

(f) [2 Pts] Rayna brings us some other models to consider. Which of the following are linear models? **Select all that apply.**

- ☐ $\hat{y}_i = \theta_0 + \theta_1 x_i$
- ☐ $\hat{y}_i = \theta_0 e^{x_i}$
- ☐ $\hat{y}_i = \theta_0 + \theta_1 x_i + \sin(\theta_2) x_i^2$
- ☐ $\hat{y}_i = \theta_0 + x_i^{\theta_1}$
- ☐ $\hat{y}_i = \theta_0 \theta_1 x_i$
- ☐ $\hat{y}_i = \theta_0 + \ln(x_i)\theta_1$
- ☐ None of the above.

5 A Sleep Descent [18 Points]

James and Rohan want to predict how well students do on an exam, y_i, using how much they sleep, x_i. They decide to use the following model:

$$\hat{y}_i = \theta_0^3 x_i^2 + e^{\theta_1}\theta_2^2 x_i,$$

where the three parameters, $\theta_0, \theta_1, \theta_2 \in \mathbb{R}$, are stored in the parameter vector $\vec{\theta} = [\theta_0, \theta_1, \theta_2]^T$.

(a) [4 Pts] James and Rohan want to use Mean Squared Error (MSE) as it is their favorite loss function. They decide to use batch gradient descent to select their optimal $\vec{\theta}$. What is the gradient vector, $\nabla_{\vec{\theta}} L$, for their choice of loss function (MSE)?

Show all work in the space below and write your final answer on the provided lines. Use $\hat{y}_i$ in your answers where possible and be sure to simplify (e.g. $2(3x)$ should be written as $6x$).

$$\nabla_{\vec{\theta}} L = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

A = _____

B = _____

C = _____

Note: There is more space for work on the next page!

(b) [2 Pts] James and Rohan realize that their dataset actually contains billions of rows. Which of the following are true for computing optimal θ? **Select all that apply.**

- ☐ Batch gradient descent is guaranteed to converge (not necessarily to the optimal parameters)
- ☐ Each iteration of batch gradient descent will be very slow
- ☐ Stochastic gradient descent will always find the optimal parameters
- ☐ Stochastic or mini-batch gradient descent may be a suitable choice
- ☐ None of the above

(c) [4 Pts] Rohan's friend suggests they use the following loss function and stochastic gradient descent instead:

$$L^{\star}(\vec{\theta}) = y_i - (\ln \theta_0 + \theta_0 \theta_1 x_i)$$

This loss function has two parameters, $\theta_0, \theta_1 \in \mathbb{R}$ which are stored in the parameter vector $\vec{\theta} = [\theta_0, \theta_1]^T$.

Pushing aside their concerns about this new loss function, James and Rohan decide to proceed and see what the optimal θ_0 and θ_1 estimates are after one iteration using the following row of data:

i	x_i	y_i
1	3	60

They pick the following starting values for θ_0 and θ_1: $\theta_0^{(0)} = 1$ and $\theta_1^{(0)} = 2$ and set learning rate $\alpha = 0.1$.

Show all work in the space below and write your final answers on the provided lines.

$\theta_0^{(1)}$ = _____

$\theta_1^{(1)}$ = _____

(d) [2 Pts] In general, which of the following statements are true? **Select all that apply.**

- ☐ Initial parameter values do not affect the gradient descent convergence.
- ☐ If a gradient descent process converged at a local minimum, then it must also have converged at the global minimum.
- ☐ The learning rate affects the speed of the gradient descent process and whether or not the model converges.
- ☐ Negative values are an appropriate choice for learning rate.
- ☐ Each update of θ may jump back and forth between two sides of the optimal $\hat{\theta}$.
- ☐ None of the above.

(e) [2 Pts] In the case that the validation and test performance are significantly worse than the training performance, which of the following statements could explain this behavior? **Select all that apply.**

- ☐ The model is underfitting. It might be possible that the model is not expressive enough to model the underlying trend in the data.
- ☐ The model is underfitting. It is possible that there is no underlying trend in our data.
- ☐ The model is overfitting. It is possible that our model is too expressive and started fitting too closely to the noise in our training data.
- ☐ The training data is not representative of the validation and test data, leading to poor performance.
- ☐ None of the above.

(f) [2 Pts] Which of the following statements about regularization are true? Assume we are using OLS. **Select all that apply.**

- ☐ L1 regularization is more likely to lead to a solution in which the coefficients are set to 0 than L2 regularization.
- ☐ L1 regularization is more sensitive to extreme outliers than L2 regularization.
- ☐ L2 regularization has a closed form solution, while L1 regularization does not.
- ☐ Introducing regularization always leads to higher performance on the training set.
- ☐ None of the above.

(g) [2 Pts] You decide to use two hyperparameters, α (the learning rate), and λ (the regularization penalty). You are considering 3 choices of α and 3 choices of λ. If you perform five-fold cross validation to choose the best pair of hyperparameters, how many validation errors would you have to calculate? **Draw a box around your final answer**.

6 Least Squares, Most Fun [8 Points]

Given the linear model $\hat{\mathbb{Y}} = \mathbb{X}\theta$ and our design matrix $\mathbb{X} \in \mathbb{R}^{n \times (p+1)}$, we have used geometric properties to show that the most optimal solution to the least squares solution is, assuming $\mathbb{X}^T\mathbb{X}$ is invertible:

$$\hat{\theta}_{OLS} = (\mathbb{X}^\top \mathbb{X})^{-1}\mathbb{X}^\top \mathbb{Y}$$

In some cases, our matrix $\mathbb{X}$ might be *wide* whereby $n << (p+1)$. In other words, we have many more features than actual observations. We can then take the right pseudo-inverse (also called Moore-Penrose Inverse) if $\mathbb{X}$ is full row rank, allowing us to calculate a least squares solution:

$$\hat{\theta}_{MP} = \mathbb{X}^\top (\mathbb{X}\mathbb{X}^\top)^{-1}\mathbb{Y}$$

For this question, you can assume that $\mathbb{X}$ contains a column of ones for the intercept, i.e. $\mathbb{X}_{:,0} = \mathbb{1}_n$.

(a) [2 Pts] During lecture, we covered the computational complexity of solving for $\hat{\theta}_{OLS}$ using the normal equation. What is the computational complexity of calculating $\hat{\theta}_{MP}$ if we use the equation above? *Use big O notation and give the tightest bound possible.*

(b) [3 Pts] Show that when $\mathbb{X}$ is full rank and square, $\hat{\theta}_{MP} = \hat{\theta}_{OLS}$. In other words, show that $\mathbb{X}^\top (\mathbb{X}\mathbb{X}^\top)^{-1} = (\mathbb{X}^\top \mathbb{X})^{-1}\mathbb{X}^\top$.

(c) [3 Pts] From class, we also know that $\sum_{i=1}^{n} e_i = 0$ when $\hat{\theta}_{OLS} = (\mathbb{X}^\top \mathbb{X})^{-1} \mathbb{X}^\top \mathbb{Y}$ and $\mathbb{X}_{:,0} = \mathbb{1}_n$. Show that this property holds when $n << (p+1)$ for $\hat{\theta}_{MP} = \mathbb{X}^\top (\mathbb{X}\mathbb{X}^\top)^{-1} \mathbb{Y}$ as well.

End of Graded Questions

7 Congratulations [0 Pts]

Congratulations! You have completed the Midterm Exam.

- **Make sure that you have written your student ID number on *each page* of the exam.** You may lose points on pages where you have not done so.
- Also ensure that you have **signed the Honor Code** on the cover page of the exam for 1 point.

[Optional, 0 pts] Draw a picture (or graph) describing your experience in Data 100.