DS-100 Midterm Exam

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Instructions:

- This midterm exam must be completed in the **80 minute time** period ending at **12:30PM**, unless you have accommodations supported by a DSP letter.
- Note that some questions have bubbles to select a choice. This means that you should only **select one choice**. Other questions have boxes. This means you should **select all that apply**.
- When selecting your choices, you must **fully shade** in the box/circle. Check marks will likely be mis-graded.
- You may use a one-sheet (two-sided) study guide.
- Work quickly through each question. There are a total of 168 points on this exam.

Honor Code:

As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. I am the person whose name is on the exam and I completed this exam in accordance with the honor code.

Signature: _____

Syntax Reference

Regular Expressions

- "^" matches the position at the beginning of string (unless used for negation " [^] ")
- "\$" matches the position at the end of string character.
- "?" match preceding literal or sub-expression 0 or 1 times. When following "+" or "*" results in non-greedy matching.
- "+" match preceding literal or sub-expression *one* or more times.
- "*****" match preceding literal or sub-expression *zero* or more times
- "." match any character except new line.

- "[]" match any one of the characters inside, accepts a range, e.g., "[a-c]".
- "() " used to create a sub-expression
- "\d" match any *digit* character. "\D" is the complement.
- "\w" match any *word* character (letters, digits, underscore). "\₩" is the complement.
- "**s**" match any *whitespace* character including tabs and newlines. \S is the complement.
- "\b" match boundary between words

Some useful re and requests package functions.

- re.findall(pattern, st) return the list of all sub-strings in st that match pattern.
- requests.get(url, auth, params, data)
 makes a GET requests with params in the
 header and data in the body.
- requests.post(url, auth, params, data)
 makes a POST requests with params in the
 header and data in the body.

Useful Pandas Syntax

```
df.loc[row_selection, col_list] # row selection can be boolean
df.iloc[row_selection, col_list] # row selection can be boolean
df.groupby(group_columns)[['colA', 'colB']].sum()
pd.merge(df1, df2, on='hi') # Merge df1 and df2 on the 'hi' column
```

pd. pivot_table (df,	<i># The input dataframe</i>
<pre>index=out_rows,</pre>	<i># values to use as rows</i>
<pre>columns=out_cols,</pre>	<i># values to use as cols</i>
<pre>values=out_values,</pre>	<i># values to use in table</i>
<pre>aggfunc="mean",</pre>	<pre># aggregation function</pre>
<pre>fill_value=0.0)</pre>	<pre># value used for missing comb.</pre>

Data Design and Bias

1. [1 Pt] Your letter grade (e.g., A+, A, ...) in a class that grades on a curve is most accurately described as what kind of data?

 \bigcirc Nominal $\sqrt{\text{Ordinal}}$ \bigcirc Quantitative \bigcirc Numerical

Solution: There is a clear ordering in the grades; however, the difference between two grades depends on the grade distribution.

2. [1 Pt] The **number** of gold medals won by each country in the 2018 Olympics is an example of what kind of data:

 \bigcirc Nominal \bigcirc Ordinal \bigcirc Qualitative $\sqrt{}$ Quantitative

Solution: While the type of medal is an ordinal variable the number of a particular medal is a quantitative variable.

- 3. A discussion leader with **32 students** in her section would like to sample a single student that is representative of the total **population of students in her section**. She enumerates her students 0 to 31 and follows one of the following procedures:
 - (a) [4 Pts] She flips a fair coin 31 times and records the number of heads. She then selects the student with the number that matches the number of heads. What type of sample has the discussion leader taken? **Select all that apply.**

 \Box Simple random sample $\sqrt{\text{Probability sample}}$ \Box Convenience sample \Box None of the above

Solution: Since we can write down the probability that each student is selected, this is an example of a probability sample. However, this is not a simple random sample as not all students are equally likely to be selected.

(b) [4 Pts] She flips a fair coin 5 times and records the sequence of heads and tails as 1's and 0's, respectively. She then selects the student whose number corresponds to the binary sequence. For example, if she flipped [1, 1, 0, 0, 1] then she would select:

$$1 * 2^{0} + 1 \times 2^{1} + 0 \times 2^{2} + 0 \times 2^{3} + 1 \times 2^{4} =$$
student 21

What type of sample has the discussion leader taken? Select all that apply.

 $\sqrt{$ Simple random sample $\sqrt{$ Probability sample \square Convenience sample \square None of the above

Solution: Since we can write down the probability that each student is selected, this is an example of a probability sample. Also, since each student (and "subset" of students) had an equal chance of being chosen, we also have a simple random sample.

- 4. Sampling True/False For each of the following select true or false:
 - (a) [1 Pt] If each element/member of the population has an equal chance of being chosen, then we have a simple random sample.

 \bigcirc True \checkmark False

Solution: False, each subset must have an equal chance of being chosen. (this is a stronger condition)

(b) [1 Pt] In cluster sampling, each cluster has an equal chance of being chosen.

```
√ True 🔿 False
```

Solution: True, by definition of cluster sampling.

(c) [1 Pt] In stratified sampling, each element of the population is a assigned to exactly one stratum.

🗸 True 🛛 🔿 False

Solution: True, by the definition of stratified sampling.

(d) [1 Pt] A small simple random sample can often be more representative of the population than a very large convenience sample.

 $\sqrt{\text{True}}$ \bigcirc False

Solution: Convenience samples are often heavily biased. In class we considered a scenario where a small carefully constructed simple random sample was less biased than a very large convenience sample.

- 5. We would like to understand the sleeping habits on university students living in campus dorms across the United States.
 - (a) [2 Pts] To keep costs down we randomly sample a subset of dorms across the United States and then construct a simple random sample of students within each of the selected dorms. This is an example of which sampling procedure:

 \bigcirc Simple random sample \bigcirc Stratified sample \checkmark Cluster sample

Solution: This is a form of cluster sampling where the clusters correspond to dorms. Within each dorm we have selected a simple random sample however a census or even stratified sample could be used.

(b) [2 Pts] Which of the following sampling procedures would ensure that we have good coverage of both male and female students within each dorm.

 \bigcirc Simple random sample $\sqrt{$ Stratified sample \bigcirc Cluster sample

Solution: A stratified sample of the students within dorm would ensure that we have good coverage of male and female students

Pandas

6. Pandas True/False

(a) [1 Pt] If the pandas DataFrame df has 10 columns, then df.iloc[:, 0:5] will return a DataFrame with 5 columns.

 $\sqrt{\text{True}}$ \bigcirc False

Solution: True pd.iloc is inclusive for the starting value and exclusive for the ending value, so it will return columns 0, 1, 2, 3, 4.

(b) [1 Pt] Assuming that len(df1) == 100 and len(df2) == 100 are both true, then df1.merge(df2, how='outer') produces at most 200 rows.

🔿 True 🧹 False

Solution: False, an outer join is the cross product of the rows and can produce up to 10,000 rows.

(c) [1 Pt] The return type of the pandas.DataFrame.groupby function can either be a DataFrame or a Series object.

 \bigcirc True \checkmark False

Solution: False. groupby returns a GroupBy object.

7. The tables **food** and **store** contain information regarding different ingredients and where to buy them. You may assume all strings are strings and numbers are floats.

This is preview of the first 5 rows of the DataFrames. You may assume it has many more rows than what is shown, with the same structure and no missing data.

food					
index	x name	color	calorie	s food_gr	roup
0	broccoli	green	25	vegetał	ole
1	chicken	pink	200	meat	
2	cheddar	yellow	350	dairy	
3	mango	yellow	40	fruit	
4	carrot	orange	50	vegetał	ole
		stor	e		
index	food_name store_name		distance	price	
0	broccoli	oli yasai		1	1.5
1	broccoli	broccoli safeway		2	2
2	cheddar trade		oes	1	4
3	mango berke		y_bowl	3	1
4	carrot	costco	-	6	5

(a) [5 Pts] Which of the following expressions returns a **Series** containing only the **names** of all the **red vegetables** in the food DataFrame? **Select all that apply.**

 $\hfill\square$ None of the above.

Solution:

□ False; it contains vegetables that may not be red
√ True; it contains only the names of all red vegetables.
□ False, it is a DataFrame
□ False; never filters out to only select red vegetables
□ False

(b) [5 Pts] **Select all** of the following expressions that generate a DataFrame containing only rows of fruit.

```
\sqrt{\text{food.set\_index("food_group").loc["fruit", :]}}
```

```
  food.where(food["food_group"] == "fruit")
```

- $\sqrt{\text{food}[\text{food}["\text{food}_group"]} == "\text{fruit"}]$
- food["food_group"] == "fruit"
- \Box None of the above.

(c) [5 Pts] Select all true statements about the following expression.

```
cal100_foods = food[food["calories"] <= 100]</pre>
nearby_stores = store[store["distance"] <= 2]</pre>
output df = call00 foods.merge(nearby stores,
                             how = "left",
                             left_on="name",
                             right_on="food_name")
     □ output_df['name'] and output_df['food_name'] are always the
        same.
     \sqrt{} output_df could contain NaN values.
     □ nearby_stores always contains the same number of rows as the output_df.
     \Box None of the above.
 Solution:
       \Box False; that is the column being merged on
       \sqrt{1} True; if none of the stores have the food item in the left table, the
         merged cols will be NaN.
       \Box False; it is a left join so it could have more rows.
       \sqrt{1} True; if every food item can be found at multiple stores, a new row is
         created for each store the food is found in. This can result in a much
         larger number of rows in the output DataFrame than food.
       \Box False:
```

(d) [4 Pts] Which of the following tables is represented by agg_df?

		calories	pr	ice				price
	food_group					food_gr	oup	
	fruit	40.0000	00 25	.500000		1	iruit	72.5
	meat	200.0000	00 14	.000000		m	eat	19.0
,	vegetable	33.3333	33 21	.666667		vegeta	able	22.0
\checkmark					\bigcirc	ca	lories	
		calories	price		food_n	ame		
	food_name			_	bro	ccoli	99	
	broccoli	25.0	27.5		Ca	arrot	1	
	carrot	50.0	11.0		chie	cken	1	
\sim	mango	40.0	72.5	6	ma	ango	20	
\bigcirc				\bigcirc				

Solution: The first table has food_groups on the index and includes both calories and price as columns since they contain integer values and will not be dropped in the groupby.

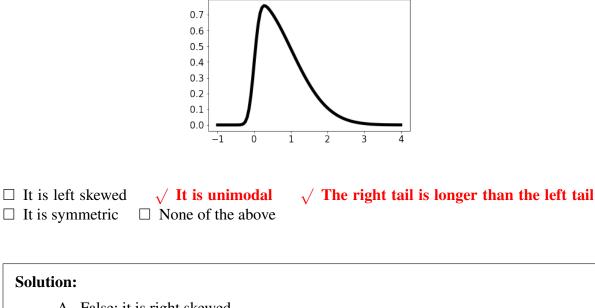
(e) [4 Pts] Which of the following expressions would generate the following table?

color	green	pink	yellow	
food_group				
fruit	40.0	78.0	40.0	
meat	10000.0	200.0	6.0	
vegetable	27.5	200.0	50.0	

- (food.groupby(["food_group", "color"])[["calories"]]
 .median())
- (food.set_index("food_group")
 .groupby("color")[["calories"]]
 .mean())

EDA and Visualization

8. [5 Pts] Which of the following claims are true for the distribution shown below? Select all that apply.



- A. False; it is right skewed
- B. True
- C. True
- D. False; it is asymmetric
- 9. [5 Pts] We wish to compare the results of kernel density estimation using a gaussian kernel and a boxcar kernel. For $\alpha > 0$, which of the following statements are true? Choose all that apply.

Gaussian Kernel:

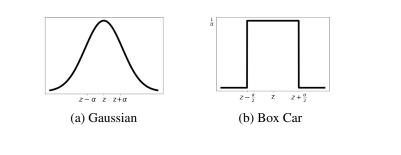
Box Car Kernel:

$$K_{\alpha}(x,z) = \frac{1}{\sqrt{2\pi\alpha^2}} \exp\left(-\frac{(x-z)^2}{2\alpha^2}\right) \qquad \qquad B_{\alpha}(x,z) = \begin{cases} \frac{1}{\alpha} & \text{if } -\frac{\alpha}{2} \le x-z \le \frac{\alpha}{2}\\ 0 & \text{else} \end{cases}$$

 $\sqrt{}$ Decreasing α for a gaussian kernel decreases the smoothness of the KDE.

 \Box The gaussian kernel is always better than the boxcar kernel for KDEs.

 \Box Because the gaussian kernel is smooth, we can safely use large α values for kernel density estimation without worrying about the actual distribution of data



\checkmark The area under the box car kernel is 1, regardless of the value of α

 \Box None of the above

Solution:

- A. True
- B. False; if the α values are not carefully selected for the gaussian kernel, the box car kernel can provide a better kernel density estimate
- C. False; if we set α too high we potentially risk including too many points in our estimate, resulting in a flatter curve.
- D. True
- 10. [5 Pts] Which of the following styles of plots are good for visualizing the distribution of a continuous variable? Choose all that apply.

 \Box Pie Charts $\sqrt{\text{Box Plots}}$ \Box Bar Plots $\sqrt{\text{Histogram}}$ \Box None of the above

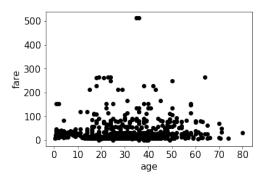
Solution:

- A. False; pie charts are bad
- B. True
- C. False; bar plots usually for nominal/ordinal data
- D. True
- 11. [2 Pts] Suppose you wish to compare the number of homes homeowners in the US own and their respective salaries. Which style of plot would be the best?

○ Scatter Plot	Overlaid Line Plots	Side by Side Box Plots	○ Stacked Bar
Plot			

Solution:

- A. False; most people own around 1-2 homes thus there will be heavy overplotting
- B. False; doesn't make sense
- C. True
- D. False; stacking is bad and bar plots won't do a good job
- 12. [5 Pts] Consider the plot below. What are some ways to improve the plot? Choose all that apply. Assume each is done individually.

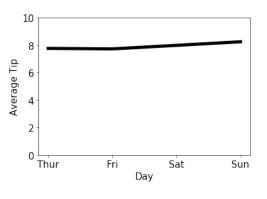


 $\sqrt{}$ Remove outliers and then plot on a different scale

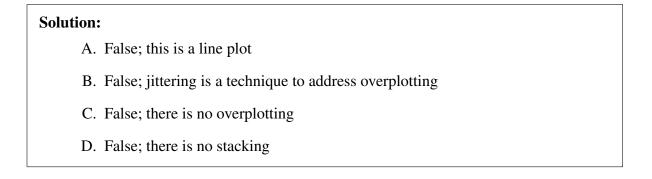
- \Box Plot as a line plot instead of a scatterplot.
- \Box Jitter the data with noise sampled from a uniform distribution of (-1, 1)
- $\sqrt{}$ Utilize transparency
- \Box None of the above

Solution:

- A. True
- B. False; for these data a line plot would be an incomprehensible web.
- C. False; simply adding a small random noise doesn't alleviate the problem at hand of markers being condensed near the x axis.
- D. True
- 13. [5 Pts] Consider the plot below which visualizes day of the week versus the average tip given in dollars. What are serious visualization errors made with this plot? Choose all that apply.



 \Box Area perception \Box Jittering \Box Overplotting \Box Stacking $\sqrt{}$ None of the above



14. True/False

(a) [1 Pt] A data scientist must always consider potential sources of bias in a given dataset.

 $\sqrt{\text{True}}$ \bigcirc False

(b) [1 Pt] It is always reasonable to drop missing values.

 \bigcirc True \checkmark False

15. Use the following dataset to answer the following questions:

```
id, diet, pulse, time, kind
1, low fat, 85, 1 min, rest
1, low fat, 85, 15 min, rest
1, low fat, 88, 30 min, rest
2, low fat, 90, 1 min, rest
2, low fat, 92, 15 min, rest
2, low fat, 93, 30 min, rest
3, low fat, 97, 1 min, rest
3, low fat, 97, 15 min, rest
```

- (a) [1 Pt] Which of the following **best** describes the format of this file?
 - Raw text
 - Tab Separated Values (TSV)
 - $\sqrt{}$ Comma Separated Values (CSV)
 - ⊖ JSON
- (b) [4 Pts] Select **all** the true statements.
 - \Box From the data available, the id seems to be a primary key.
 - $\sqrt{}$ There appear to be no missing values.
 - \Box There are nested records.
 - \Box None of the above.
- 16. [5 Pts] Select all the true statements about the following XML file:

```
1 < \text{email} >
2
            <to>Mr. Garcia
3
                     <body>Hello there! How are we today?</to>
4
            </body>
5 < /email >
6 < \text{email} >
7
            <to>Mr. Garcia
8
                     <body>Hello there! How are we today?</to>
9
            </body>
10 < /email >
```

- \Box This XML file is correctly formatted.
- $\sqrt{}$ Tags are not properly nested.
- $\sqrt{}$ This XML file is missing one root node that contains all the other nodes
- $\sqrt{}$ The email tag on lines 1, 5, 6 and 10 should not have spaces between $\{<, >\}$ and tag name.
- \Box None of the above are true.
- 17. Use the following JSON file classes.json printed below:

```
1 [{
2
       "Prof": "Gonzalez",
       "Classes": [ "CS186",
3
4
           {
                "Name": "Data100",
5
                "Year": [2017, 2018]
6
7
           }],
       "Tenured": false
8
9 },
10 {
       "Prof": "Nolan",
11
       "Classes": ["Stat133", "Stat153", "Stat198", "Data100"],
12
13
       "Tenured": true
14 }]
```

(a) [5 Pts] Select all the true statements.

- $\sqrt{}$ This JSON file is correctly formatted.
- □ The Classes list defined on line 3 contains strings and dictionaries which is not permitted.
- \Box The dates 2017 and 2018 on lines 6 should be quoted.
- □ the dictionary keys (e.g., "Prof", "Classes") should not be quoted.
- \Box None of the above statements are true.
- (b) [3 Pts] What would be the output of the following block of code:

 $\bigcirc 1 \bigcirc 2 \bigcirc 4 \checkmark 5 \bigcirc$ None of the above.

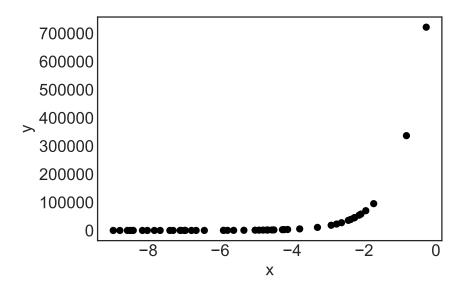
18. [6 Pts] Which data formats would be well suited for nested data? Select all that apply. $\Box *.csv \quad \sqrt{*.xml} \quad \Box *.py \quad \sqrt{*.json} \quad \Box *.tsv \quad \Box$ None of

the above.

- 19. [6 Pts] Which of the following are reasonable motivations for applying a log transformation? Select all that apply:
 - \Box Perform dimensionality reduction on the data.
 - $\sqrt{}$ To help straighten relationships between pairs of variables.
 - \Box Remove missing values.
 - □ Bring data distribution closer to random sampling.
 - $\sqrt{}$ To help visualize highly skewed distributions.
 - \Box None of the above.
- 20. [4 Pts] Which of the of the following record is the most **coarse** grained?

```
○ {"Location": "Downtown Berkeley", "avg_income": 83000}
○ {"Location": "Los Angeles, CA", "avg_income": 75042}
○ {"Location": "Bay Area, CA", "avg_income": 73042}
√ {"Location": "California", "avg_income": 50001}
```

- 21. [4 Pts] Which of the following transformations would be best suited to linearize the relationship shown in the plot below? Note that all y > 0.:
 - Plotting $\log(y)$ vs $\log(x)$. ○ Plotting $\log(y)$ vs x. ○ Plotting $\exp(y)$ vs $\exp(x)$. ○ Plotting $\exp(y)$ vs $\log(x)$. ○ Plotting $\log(y)$ vs $\log(\log(x))$



Regular Expressions and String Manipulation

22. What would the following lines of code return? There are no spaces in any of the strings.

(a) [3 Pts] re.findall(r"\..*", "VIXX-Error.mp3.bak")

```
() [] ('bak'] ('.bak'] ('.mp3', '.bak']
√ ['.mp3.bak'] ('VIXX-Error.mp3.bak']
```

Solution: This is a regular expression search for a dot followed by anything until the end of the string. Since the search is greedy, it finds the longest match possible, which is ".mp3.bak"

(b) [3 Pts] re.findall(r"[cat|dog]", "bobcat")

○ [] ○ ['cat'] ○ ['c', 'a', 't'] ○ ['o', 'cat'] √ ['o', 'c', 'a', 't'] ○ None of the above

Solution: This is a single-character search for any of the characters in the character class. The first match is the "o" followed by "c", "a", and "t".

(c) [3 Pts] re.findall(r"a?p*[le]\$", "apple")

○ [] √ ['e'] ○ ['appl'] ○ ['appe'] ○ ['a', 'pp', 'l', 'e'] ○ None of the above

Solution: The search starts at the end of the string or an "l" or an "e", which matches the "e" in 'apple'. The regex engine then looks for a "p", but fails to find one next to the "e", so it continues by looking for an "a" next to the "e", which it also doesn't find, ending the search.

Solution: The regular expression consists of two sub-expressions. The first is for closing tags and the second is for single tags.

23. [9 Pts] On which of the following words would the regular expression $r"^{w[^p].*r"}$ return a match (on part or all of the word) instead of None? Choose all that apply.

 $\square \text{ sporous } \square \text{ sooloos } \sqrt{\text{ murdrum }} \sqrt{\text{ repaper }} \sqrt{\text{ hydroaviation }} \sqrt{\text{ defendress }} \sqrt{\text{ gourmet }} \square \text{ level } \sqrt{\text{ redder }}$

24. [5 Pts] Which regular expression would match part or all of the words on the left but NONE the ones on the right? Choose all that apply

		flossy beefin	baronet oriole	
			scupper	
√ ^.{5}[^e]?\$	□ ^.+[^e]?\$	□ [a-z]5[^e]?	2\$ √ [fh]	\Box None of the Above

Modeling and Estimation

25. Let x_1, \ldots, x_n denote any collection of numbers with average $\overline{x} = \frac{1}{n} \sum_{i=1}^n x_i$.

(a) [3 Pts] $\sum_{i=1}^{n} (x_i - \overline{x})^2 \le \sum_{i=1}^{n} (x_i - c)^2$ for all c.

 $\sqrt{\text{True}}$ \bigcirc False

Solution: The mean minimizes the square-error loss.

- (b) [3 Pts] $\sum_{i=1}^{n} |x_i \overline{x}| \le \sum_{i=1}^{n} |x_i c|$ for all *c*.
 - \bigcirc True \checkmark False

Solution: The median minimizes the absolute loss, and in general the median is not equal to the mean.

26. Consider the following loss function based on data x_1, \ldots, x_n :

$$\ell(\mu, \sigma) = \log(\sigma^2) + \frac{1}{n\sigma^2} \sum_{i=1}^n (x_i - \mu)^2.$$

- (a) [5 Pts] Which estimator $\hat{\mu}$ is a minimizer for μ , i.e. satisfies $\ell(\hat{\mu}, \sigma^2) \leq \ell(\mu, \sigma^2)$ for any μ, σ ?
 - $\bigcirc \widehat{\mu} = 0$ $\checkmark \widehat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$ $\bigcirc \widehat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i + \log\left(\frac{1}{n} \sum_{i=1}^{n} x_i\right)^2$ $\bigcirc \widehat{\mu} = \frac{1}{n\sigma^2} \sum_{i=1}^{n} x_i + \log\left(\sigma^2\right)$ $\bigcirc \widehat{\mu} = \operatorname{median}(x_1, \dots, x_n).$

Solution: The mean minimizes the square-error loss.

(b) [10 Pts] Which of the following is the result of solving $\frac{\partial \ell}{\partial \sigma} = 0$ for σ (for fixed μ)? Show your work in the box below.

$$\bigcirc \sigma = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2.$$

$$\checkmark \sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2}.$$

$$\bigcirc \sigma = \frac{2}{n} \sum_{i=1}^{n} (\mu - x_i).$$

$$\bigcirc \sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - x_j)^2}.$$

Solution: Note $\log \sigma^2 = 2 \log \sigma$, so

$$0 = \frac{\partial \ell}{\partial \sigma} = \frac{2}{\sigma} - \frac{2}{n\sigma^3} \sum_{i=1}^n (x_i - \mu)^2.$$

Rearranging, we obtain

$$\sigma^{2} = \frac{1}{n} \sum_{i=1}^{n} (x_{i} - \mu)^{2}$$

27. [10 Pts] Consider the following loss function based on data x_1, \ldots, x_n with mean \overline{x} :

$$\ell(\beta) = \log \beta + \frac{\overline{x}}{\beta} + \frac{1}{n} \sum_{i=1}^{n} e^{-x_i/\beta}$$

Given an estimate $\beta^{(t)}$, write out the update $\beta^{(t+1)}$ after one iteration of gradient descent with step size α . Show your work in the box below.

Solution: The update is

 $\beta^{(t+1)} \leftarrow \beta^{(t)} - \alpha \,\ell'(\beta^{(t)}),$

where

$$\ell'(\beta) = \frac{1}{\beta} \left(1 - \frac{\overline{x}}{\beta} + \frac{1}{n\beta} \sum_{i=1}^{n} x_i e^{-x_i/\beta} \right)$$
$$= \frac{1}{\beta} - \frac{\overline{x}}{\beta^2} + \frac{1}{n\beta^2} \sum_{i=1}^{n} x_i e^{-x_i/\beta}$$

Alternate notation:

$$\beta^{(t+1)} \leftarrow \beta^{(t)} - \alpha \left. \frac{\partial \ell}{\partial \beta} \right|_{\beta = \beta^{(t)}}$$

With everything substituted in:

$$\beta^{(t+1)} \leftarrow \beta^{(t)} - \alpha \left(\frac{1}{\beta^{(t)}} - \frac{\overline{x}}{\beta^{(t)^2}} + \frac{1}{n\beta^{(t)^2}} \sum_{i=1}^n x_i e^{-x_i/\beta^{(t)}} \right)$$