

Extra Probability Problems

Name:

- Let p denote the probability that a particular item A appears in a simple random sample (SRS). Suppose we collect 5 independent simple random samples, i.e., each SRS is obtained by drawing from the entire population. Let X denote the random variable for the total number of times that A appears in these 5 samples. What is the expected value of X , i.e., $\mathbb{E}[X]$? Your answer should be in terms of p .
 - What is $\text{Var}(X)$? Again, your answer should be in terms of p .
- Show that if two random variables X and Y are independent, then $\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y)$. You may not use the fact that $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ if X and Y are independent. Instead, use linearity of expectations and the definition of variance. *Hint*: If two random variables are independent, then their covariance is 0 and $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$.
- Consider rolling (independently) one fair six-sided die and one loaded six-sided die. Let X_1 and X_2 denote, respectively, the number of spots from one roll of the fair die and one roll of the loaded die. Suppose the distribution for the loaded die is

$$\begin{aligned}\Pr(X_2 = 1) = \Pr(X_2 = 2) &= \frac{1}{16} \\ \Pr(X_2 = 3) = \Pr(X_2 = 4) &= \frac{3}{16} \\ \Pr(X_2 = 5) = \Pr(X_2 = 6) &= \frac{4}{16}.\end{aligned}$$

Let $Y = X_1X_2$ denote the product of the two numbers of spots.

- What is the expected value of Y .
- What is the variance of Y .
- Estimate the sampling distribution of Y by simulating 10,000 rolls of the pair of dice. Provide a graphical display of the distribution. Compare the mean and variance from this estimate to the values you computed above.