Data 100, Fall 2021

Discussion #1 Solutions

Welcome to Data 100!

Linear Algebra Fundamentals

1. Linear algebra is what powers linear regression, logistic regression, and PCA (concepts we will be studying in this course). Moving forward, you will need to understand how matrix-vector operations work. That is the aim of this problem.

   Fernando, Alvin, and Kobe are shopping for fruit at Berkeley Bowl. Berkeley Bowl, true to its name, only sells fruit bowls. A fruit bowl contains some fruit and the price of a fruit bowl is the total price of all of its individual fruit.

   Berkeley Bowl has apples for \$2, bananas for \$1, and cantaloupes for \$4. (expensive!). The price of each of these can be written in a vector:

$$\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

   Berkeley Bowl sells the following fruit bowls:

   1. 2 of each fruit
   2. 5 apples and 8 bananas
   3. 2 bananas and 3 cantaloupes
   4. 10 cantaloupes

   (a) Define a matrix B such that

$$B\vec{v}$$

   evaluates to a length 4 column vector containing the price of each fruit bowl. The first entry of the result should be the cost of fruit bowl 1, the second entry the cost of fruit bowl 2, etc.

   Solution:

$$B = \begin{bmatrix} 2 & 2 & 2 \\ 5 & 8 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 10 \end{bmatrix}$$

(b) Fernando, Alvin, and Kobe make the following purchases:

- Fernando buys 2 fruit bowl 1s and 1 fruit bowl 2.
- Alvin buys 1 of each fruit bowl.
- Kobe buys 10 fruit bowl 4s (he really like cantaloupes).

Define a matrix A such that the matrix expression

$$AB\vec{v}$$

evaluates to a length 3 column vector containing how much each of them spent. The first entry of the result should be the total amount spent by Fernando, the second entry the amount sent by Alvin, etc.

Solution:

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

(c) Let's suppose Berkeley Bowl changes their fruit prices, but you don't know what they changed their prices to. Fernando, Alvin, and Kobe buy the same quantity of fruit baskets and the number of fruit in each basket is the same, but now they each spent these amounts:

$$\vec{x} = \begin{bmatrix} 80 \\ 80 \\ 100 \end{bmatrix}$$

In terms of A, B, and $\vec{x}$, determine $\vec{v_2}$ (the new prices of each fruit).

Solution:

We know that $\vec{x} = AB\vec{v_2}$ from the previous part. To solve for $\vec{v_2}$ we need to left-multiply both sides of the above equation by $(AB)^{-1}$. Doing so yields $\vec{v_2} = (AB)^{-1}\vec{x}$.

This assumes that the product AB is invertible. If you work out the product, you will see that it is full rank and thus invertible.