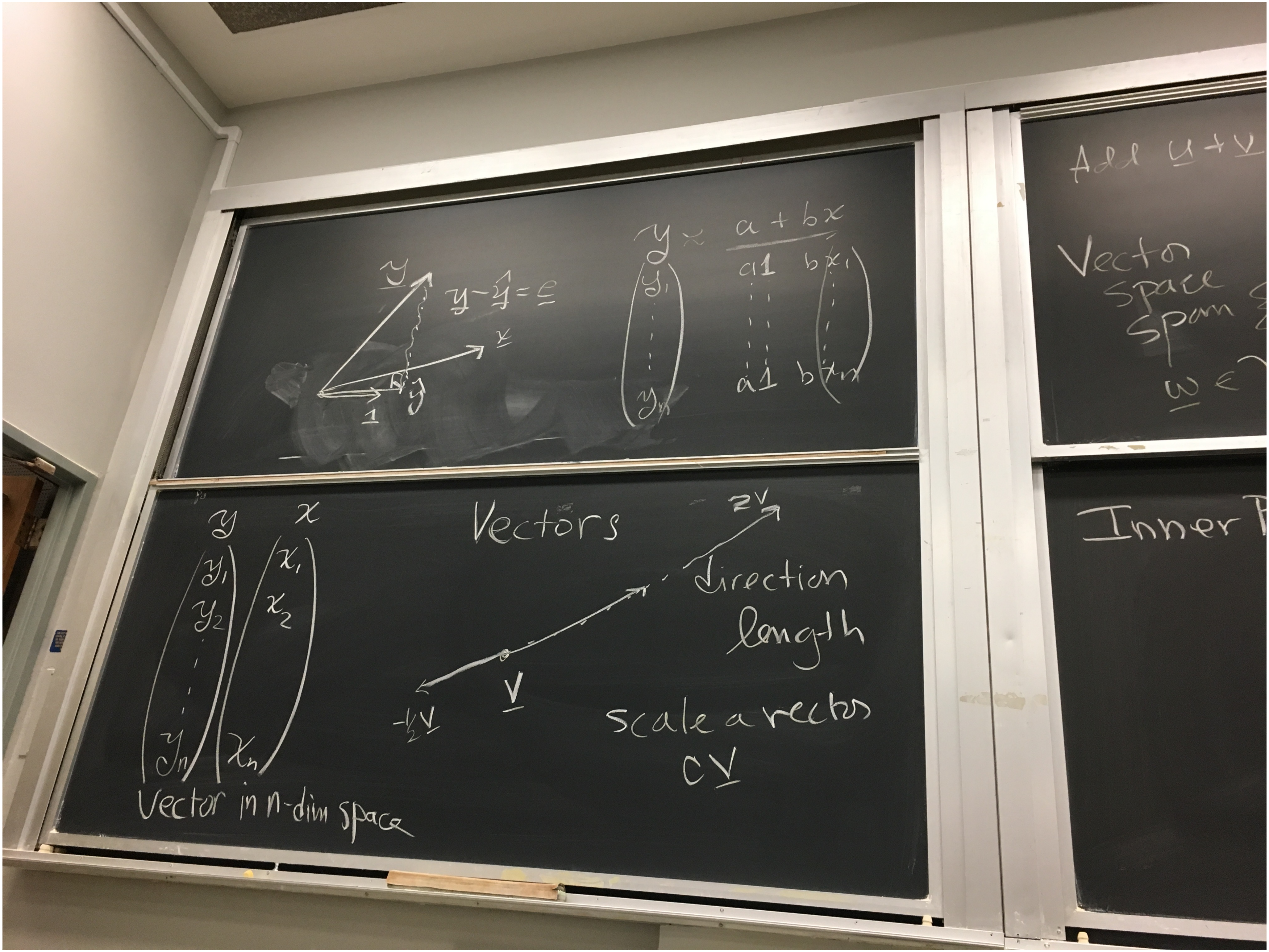
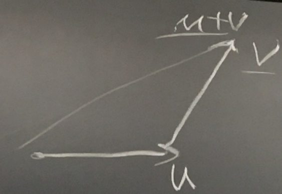

y - ŷ = e
y ≈ a + bx
a1 b x1
a1 b xn
Vectors
2V
direction
length
V
-½V
scale a vector
cV
y x
y1 x1
y2 x2
yn xn
Vector in n-dim space
Add u + v
Vector
Space
Span
w ∈
Inner P

Add $\underline{u}+\underline{v}$



Vector
Space
Span $\{\underline{v}_1, \underline{v}_2, \dots, \underline{v}_d\}$ $\mathcal{V}$

$\underline{w} \in \mathcal{V}$ $\quad w = c_1\underline{v}_1 + c_2\underline{v}_2 + \cdots + c_d\underline{v}_d$

Inner Product $\quad \underline{u}\cdot\underline{v} = u_1v_1 + u_2v_2 + \cdots + u_dv_d$

Length $\quad \sqrt{\underline{v}\cdot v} = \|\underline{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots v_d^2}$

Pythagorean

Orthogonal Vectors $\quad \underline{u}\cdot\underline{v} = 0$

Connection to fitting a line w/ L_2 loss)

$$\sum_i \left(y_i - (a + b x_i)\right)^2 \quad \text{min wrt } a, b$$

$$\begin{array}{ccccc} y_1 & - & a1 & - & bx_1 \\ \vdots & & \vdots & & \vdots \\ y_n & & a1 & & bx_n \end{array}$$

$$\|\underline{y} - (a\underline{1} + b\underline{x})\|^2$$

min the distance between $\underline{y}$ and $\operatorname{span}\{\underline{1}, \underline{x}\}$

$X = [\underline{1}, \underline{x}]$

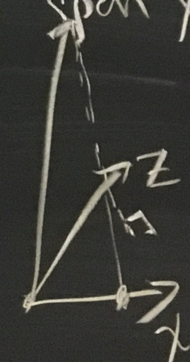
$\tilde{X} = [\underline{1}, \underline{x}, \underline{z}]$

y project onto span X

project onto span $\tilde{X}$

$$\hat{\beta} = [X^t X]^{-1} X^t y$$

$$\tilde{\beta} = [\tilde{X}^t \tilde{X}]^{-1} \tilde{X}^t y$$



$\hat{y}$ is the projection of y into span$\{X\}$

e is orthogonal to $\hat{y}$

$$y = \hat{y} + e$$
$$= X\hat{\beta} + e$$
$$X^t y = X^t X \hat{\beta} + X^t e$$

$\hat{\beta}_{(p+1)\times 1}$

$$\underline{1} \cdot e = 0$$
$$\underline{x}_1 \cdot e = 0$$
$$\vdots$$
$$x_p \cdot e = 0$$

$$X^t y = X^t X \hat{\beta}$$

$$(X^t X)^{-1} X^t y = \hat{\beta}$$ Normal equations

collinearity

$p > n$